Capital Budgeting

[Compiled from (a) Van Horne & Wachowicz, and (b) Gitman]

What is Capital Budgeting?

The process of identifying, analyzing, and selecting investment projects whose returns (cash flows) are expected to extend beyond one year.

The Capital Budgeting Process

- Generate investment proposals consistent with the firm's strategic objectives.
- Estimate after-tax incremental operating cash flows for the investment projects.
- Evaluate project incremental cash flows.

The Capital Budgeting Process

- Select projects based on a value-maximizing acceptance criterion.
- Reevaluate implemented investment projects continually and perform post-audits for completed projects.

Classification of Investment Project Proposals

1. New products or expansion of existing products
2. Replacement of existing equipment or buildings
3. Research and development
4. Exploration
5. Other (e.g., safety or pollution related)

Screening Proposals and Decision Making

1. Section Chiefs
2. Plant Managers
3. VP for Operations
4. Capital Expenditures Committee
5. President
6. Board of Directors

Advancement to the next level depends on cost and strategic importance.

Example of a Washing Machine

Brajesh is head of a joint family consisting of 10 people. There is tremendous family pressure on him to purchase automatic heavy duty washing machine. He is considering purchase of a high class washing machine. The washing machine will cost AED 40,000. He enquires and finds that they wash clothes almost every day. He also finds out that this will help him save AED 40 per day [paid to servant, etc.]. The machine has a life of 3 years. Should he go for the purchase?

Example …

Year	0	1	2	3
Opeartional Income/Expense	-40000	13200	13200	13200
Cumulative	-40000	-26800	-13600	-400
So, he rejects				

Washing Machine Continues …

One of the family members is a smart accounts student. She suggests that why don't you so show this as part of your business and then adjust the depreciation. She also finds that it can be procured from the market at AED 36000/- (basically 10% discount). She also suggests that we can use straight line depreciation with zero salvage value?

Example …

Year	0	1	2	3
Incremental Operation Profits	-36000	13200	13200	13200
Less Depreciation		12000	12000	12000
Incremental PBT		1200	1200	1200
tax @36%		432	432	432
Incremental PAT		768	768	768
Cash Flow from Machine	-36000	12768	12768	12768
Cumulative	-36000	-23232	-10464	2304

- What is the payback period of the machine?
- Around 2 years 10 months
- But,
- We have not considered the ‘Time Value of Money’
- Brajesh is in the business of Agri-products business and the cost of capital of his firm is 15%
- How should he evaluate the business?
- Concept of IRR and NPV

Cash Flow from Machine	-36000	12768	12768	12768
Internal Rate of Return (IRR)	3.17%			
Present Value of Cash Flows	-36000	11103	9654	8395
Net Present Value (NPV)	-6848			

Estimating After-Tax Incremental Cash Flows

Basic characteristics of relevant project flows

- ☑ Cash (not accounting income) flows
- ☑ Operating (not financing) flows
- ☑ After-tax flows
- ☑ Incremental flows

Estimating After-Tax Incremental Cash Flows

Principles that must be adhered to in the estimation

- ☑ ***Ignore*** sunk costs
- ☑ ***Include*** opportunity costs
- ☑ ***Include*** project-driven changes in working capital
- ☑ ***Include*** effects of inflation

Tax Considerations and Depreciation

- Depreciation represents the systematic allocation of the cost of a capital asset over a period of time for financial reporting purposes, tax purposes, or both.

  - Generally, profitable firms prefer to use an accelerated method for tax reporting purposes such as MACRS or DDB (WDV) method instead of the Straight Line Method.

Depreciation and the MACRS Method

- Everything else equal, the greater the depreciation charges, the lower the taxes paid by the firm.
- Depreciation is a non-cash expense.
- In MACRS (Modified Accelerated Cash Recovery System) type of depreciation, assets are depreciated (MACRS) on one of eight different property classes.

MACRS Sample Schedule

Recovery	Property Class		
Year	3-Year	5-Year	7-Year
1	33.33%	20.00%	14.29%
2	44.45	32.00	24.49
3	14.81	19.20	17.49
4	7.41	11.52	12.49
5		11.52	8.93
6		5.76	8.92
7			8.93
8			4.46

Depreciable Basis

In tax accounting, the fully installed cost of an asset. This is the amount that, by law, may be written off over time for tax purposes.

Depreciable Basis =

Cost of Asset + Capitalized Expenditures

Capitalized Expenditures

Capitalized Expenditures are expenditures that may provide benefits into the future and therefore are treated as capital outlays and not as expenses of the period in which they were incurred.

Examples: Shipping and installation

EXAMPLE

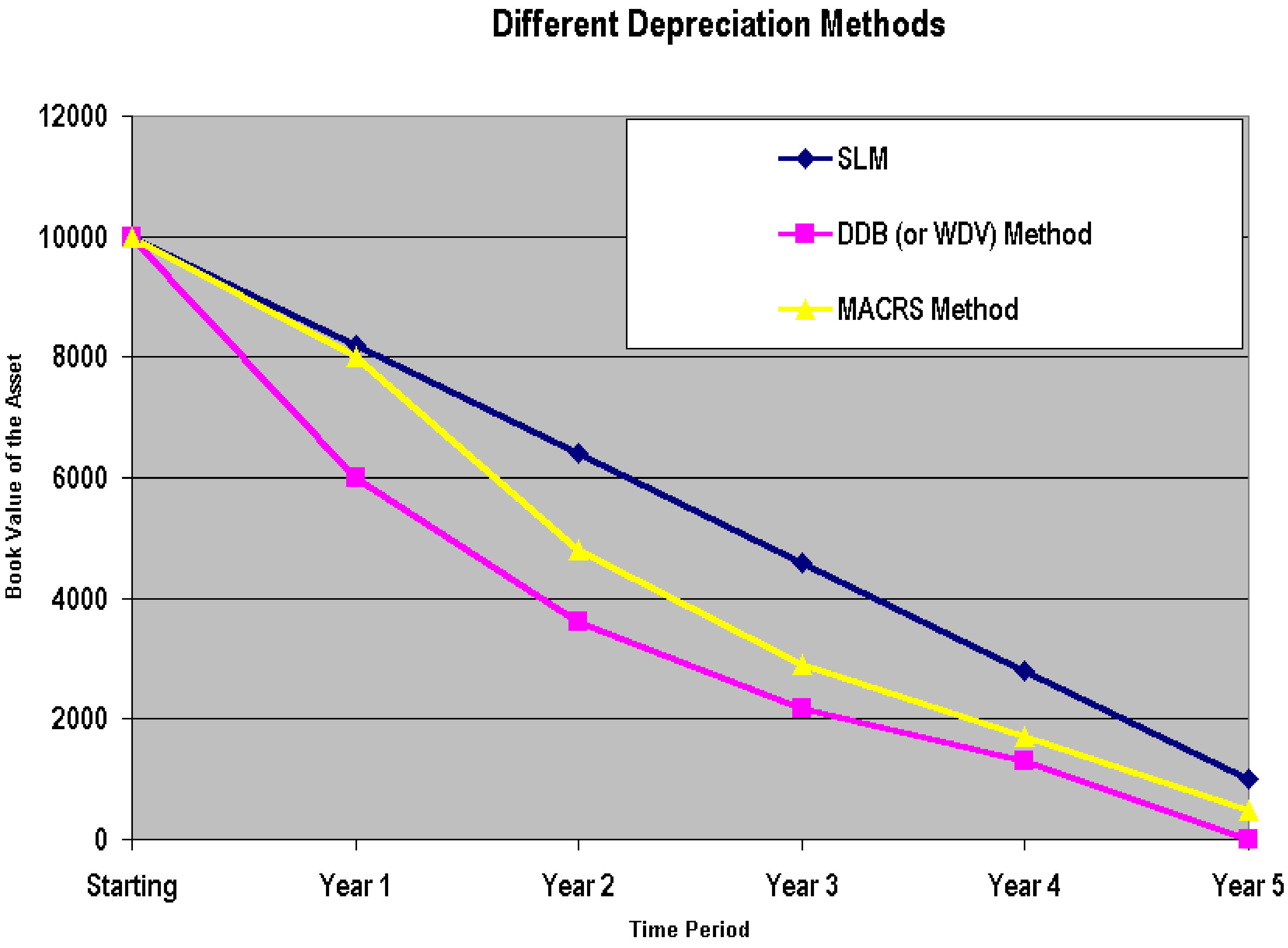
Plant Value	9500
Installation Costs	500
Depreciable Value	10000
Salvange Value	1000
Time Period	5

Depreciation Types

- SLM is Staight Line Method
- MACRS is Modified Cash Recovery System
- DDB is Double Declining Balance Method

Depreciation Type	Year 1	Year 2	Year 3	Year 4	Year 5
SLM	1800	1800	1800	1800	1800
DDB (or WDV) Method	4000	2400	1440	864	1296
MACRS Method	2000	3200	1900	1200	1200

Book Values	Starting	Year 1	Year 2	Year 3	Year 4	Year 5
SLM	10000	8200	6400	4600	2800	1000
DDB (or WDV) Method	10000	6000	3600	2160	1296	0
MACRS Method	10000	8000	4800	2900	1700	500


Different Depreciation Methods
12000
10000
8000
6000
4000
2000
0
Book Value of the Asset
SLM
DDB (or WDV) Method
MACRS Method
Starting
Year 1
Year 2
Year 3
Year 4
Year 5
Time Period

Sale or Disposal of a Depreciable Asset

- Generally, the sale of a “capital asset” generates a capital gain (asset sells for more than book value) or capital loss (asset sells for less than book value).

Calculating the Incremental Cash Flows

- Initial cash outflow -- the initial net cash investment.
- Interim incremental net cash flows -- those net cash flows occurring after the initial cash investment but not including the final period's cash flow.
- Terminal-year incremental net cash flows -- the final period's net cash flow.

Initial Cash Outflow

a)		*Cost of "new" assets*
b)	+	Capitalized expenditures
c)	+ (-)	Increased (decreased) NWC
d)	-	Net proceeds from sale of "old" asset(s) if replacement
e)	+ (-)	Taxes (savings) due to the sale of "old" asset(s) if replacement
f)	=	Initial cash *outflow*

Incremental Cash Flows

a)		Net incr. (decr.) in operating revenue less (plus) any net incr. (decr.) in operating expenses, excluding depr.
b)	- (+)	Net incr. (decr.) in tax depreciation
c)	=	Net change in income before taxes
d)	- (+)	Net incr. (decr.) in taxes
e)	=	Net change in income after taxes
f)	+ (-)	Net incr. (decr.) in tax depr. charges
g)	=	Incremental net cash flow for period

Terminal-Year Incremental Cash Flows

a)		Calculate the incremental net cash flow for the terminal period
b)	+ (-)	Salvage value (disposal/reclamation costs) of any sold or disposed assets
c)	- (+)	Taxes (tax savings) due to asset sale or disposal of "new" assets
d)	+ (-)	Decreased (increased) level of "net" working capital
e)	=	Terminal year incremental net cash flow

Example of an Asset Expansion Project

Sameer Baskets (SB) is considering the purchase of a new basket weaving machine. The machine will cost $50,000 plus $20,000 for shipping and installation and falls under the 3-year MACRS class. NWC will rise by $5,000. Sameer forecasts that revenues will increase by $110,000 for each of the next 4 years and will then be sold (scrapped) for $10,000 at the end of the fourth year, when the project ends. Operating costs will rise by $70,000 for each of the next four years. SB is in the 40% tax bracket.

Initial Cash Outflow

a)		$50,000	
b)	+	20,000	
c)	+	5,000	
d)	-	0	(not a replacement)
e)	+ (-)	0	(not a replacement)
f)	=	$75,000*	

Incremental Cash Flows

		Year 1	Year 2	Year 3	Year 4
a)		$40,000	$40,000	$40,000	$40,000
b)	-	23,331	31,115	10,367	5,187
c)	=	$16,669	$ 8,885	$29,633	$34,813
d)	-	6,668	3,554	11,853	13,925
e)	=	$10,001	$ 5,331	$17,780	$20,888
f)	+	23,331	31,115	10,367	5,187
g)	=	$33,332	$36,446	$28,147	$26,075

Terminal-Year Incremental Cash Flows

a)		$26,075	The incremental cash flow from the previous slide in Year 4.
b)	+	10,000	Salvage Value.
c)	-	4,000	.40*($10,000 - 0) Note, the asset is fully depreciated at the end of Year 4.
d)	+	5,000	NWC - Project ends.
e)	=	$37,075	Terminal-year incremental cash flow.

Summary of Project Net Cash Flows

Asset Expansion

Year 0	Year 1	Year 2	Year 3	Year 4
-$75,000*	$33,332	$36,446	$28,147	$37,075

Example of an Asset Replacement Project

Let us continue the problem in Slide 21. Assume that the previous asset expansion project is actually an asset replacement project. The original basis of the machine was $30,000 and depreciated using straight-line over five years ($6,000 per year). The machine has two years of depreciation and four years of useful life remaining. SB can sell the current machine for $6,000. The new machine will not increase revenues (remain at $110,000) but it decreases operating expenses by $10,000 per year (old = $80,000). NWC will rise to $10,000 from $5,000 (old).

Initial Cash Outflow

a)		$50,000	
b)	+	20,000	
c)	+	5,000	
d)	-	6,000	(sale of "old" asset)
e)	-	2,400	<----
f)	=	$66,600	

(tax savings from loss on sale of "old" asset)

Calculation of the Change in Depreciation

		Year 1	Year 2	Year 3	Year 4
a)		$23,331	$31,115	$10,367	$ 5,187
b)	-	6,000	6,000	0	0
c)	=	$17,331	$25,115	$10,367	$ 5,187

a) Represent the depreciation on the "new" project.

b) Represent the remaining depreciation on the "old" project.

c) Net change in tax depreciation charges.

Incremental Cash Flows

		Year 1	Year 2	Year 3	Year 4
a)		$10,000	$10,000	$10,000	$10,000
b)	-	17,331	25,115	10,367	5,187
c)	=	$ -7,331	-$15,115	$ -367	$ 4,813
d)	-	-2,932	-6,046	-147	1,925
e)	=	$ -4,399	$ -9,069	$ -220	$ 2,888
f)	+	17,331	25,115	10,367	5,187
g)	=	$12,932	$16,046	$10,147	$ 8,075

Terminal-Year Incremental Cash Flows

a)		$ 8,075	The incremental cash flow from the previous slide in Year 4.
b)	+	10,000	Salvage Value.
c)	-	4,000	(.40)*($10,000 - 0). Note, the asset is fully depreciated at the end of Year 4.
d)	+	5,000	Return of “added” NWC.
e)	=	$19,075	Terminal-year incremental cash flow.

Summary of Project Net Cash Flows

Asset Expansion

Year 0	Year 1	Year 2	Year 3	Year 4
-$75,000	$33,332	$36,446	$28,147	$37,075

Asset Replacement

Year 0	Year 1	Year 2	Year 3	Year 4
-$66,600	$12,933	$16,046	$10,147	$19,075

Capital Budgeting Techniques

Project Evaluation: Alternative Methods

- Payback Period (PBP)
- Internal Rate of Return (IRR)
- Net Present Value (NPV)
- Profitability Index (PI)

Independent Project

- For this project, assume that it is independent of any other potential projects that *Sameer Baskets* may undertake.

- Independent -- A project whose acceptance (or rejection) does not prevent the acceptance of other projects under consideration.

Payback Period (PBP)

0	1	2	3	4	5
-40 K	10 K	12 K	15 K	10 K	7 K

PBP is the period of time required for the cumulative expected cash flows from an investment project to equal the initial cash outflow.

Payback Solution (#1)

0	1	2	*3* (a)	4	5
-40 K (-b)	10 K	12 K	15 K	*10 K* (d)	7 K
	10 K	22 K	*37 K* (c)	47 K	54 K

Cumulative Inflows

PBP = a + (b - c) / d
= 3 + (40 - 37) / 10
= 3 + (3) / 10
= 3.3 Years

Payback Solution (#2)

0	1	2	3	4	5
-40 K	10 K	12 K	15 K	10 K	7 K
-40 K	-30 K	-18 K	-3 K	7 K	14 K

Cumulative Cash Flows

PBP = 3 + (3K) / 10K
= 3.3 Years

Note: Take absolute value of last negative cumulative cash flow value.

PBP Acceptance Criterion

The management of *Sankar Baskets* has set a maximum PBP of 3.5 years for projects of this type.

Should this project be accepted?

Yes! The firm will receive back the initial cash outlay in less than 3.5 years. [3.3 Years < 3.5 Year Max.]

PBP Strengths and Weaknesses

Strengths:

- Easy to use and understand
- Can be used as a measure of liquidity
- Easier to forecast ST than LT flows

Weaknesses:

- Does not account for TVM
- Does not consider cash flows beyond the PBP
- Cutoff period is subjective

Internal Rate of Return (IRR)

IRR is the discount rate that equates the present value of the future net cash flows from an investment project with the project's initial cash outflow.

$$ICO = \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_n}{(1+IRR)^n}$$

IRR Solution

$$\$40{,}000 = \frac{\$10{,}000}{(1+IRR)^1} + \frac{\$12{,}000}{(1+IRR)^2} + \frac{\$15{,}000}{(1+IRR)^3} + \frac{\$10{,}000}{(1+IRR)^4} + \frac{\$7{,}000}{(1+IRR)^5}$$

Find the interest rate (*IRR*) that causes the discounted cash flows to equal $40,000.

IRR Solution (Try 10%)

$$40,000 = $10,000(PVIF$_{10\%,1}$) + $12,000(PVIF$_{10\%,2}$) + $15,000(PVIF$_{10\%,3}$) + $10,000(PVIF$_{10\%,4}$) + $ 7,000(PVIF$_{10\%,5}$)

$40,000 = $10,000(.909) + $12,000(.826) + $15,000(.751) + $10,000(.683) + $ 7,000(.621)

$40,000 = $9,090 + $9,912 + $11,265 + $6,830 + $4,347

= $41,444 [*Rate is too low!!*]

IRR Solution (Try 15%)

$\$40{,}000 = \$10{,}000(PVIF_{15\%,1}) + \$12{,}000(PVIF_{15\%,2}) + \$15{,}000(PVIF_{15\%,3}) + \$10{,}000(PVIF_{15\%,4}) + \$\ 7{,}000(PVIF_{15\%,5})$

$\$40{,}000 = \$10{,}000(.870) + \$12{,}000(.756) + \$15{,}000(.658) + \$10{,}000(.572) + \$\ 7{,}000(.497)$

$\$40{,}000 = \$8{,}700 + \$9{,}072 + \$9{,}870 + \$5{,}720 + \$3{,}479$

$= \$36{,}841$ [*Rate is too high!!*]

IRR Solution (Interpolate)

.05	X	.10	$41,444	$1,444	$4,603
		IRR	$40,000		
		.15	$36,841		

$$\frac{X}{.05} = \frac{\$1,444}{\$4,603}$$

IRR Solution (Interpolate)

.05	X	.10	$41,444	$1,444	$4,603
		IRR	$40,000		
		.15	$36,841		

$$X = \frac{(\$1{,}444)(0.05)}{\$4{,}603} \qquad X = .0157$$

IRR = .10 + .0157 = .1157 or 11.57%

IRR Acceptance Criterion

The management of *Sankar Baskets* has determined that the hurdle rate is 13% for projects of this type.

Should this project be accepted?

No! The firm will receive 11.57% for each dollar invested in this project at a cost of 13%. [IRR < Hurdle Rate]

IRR Strengths and Weaknesses

Strengths:

- Accounts for TVM
- Considers all cash flows
- Less subjectivity

Weaknesses:

- Assumes all cash flows reinvested at the IRR
- Difficulties with project rankings and Multiple IRRs

Net Present Value (NPV)

NPV is the present value of an investment project's net cash flows minus the project's initial cash outflow.

$$NPV = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots + \frac{CF_n}{(1+k)^n} - ICO$$

NPV Solution

Sankar Baskets has determined that the appropriate discount rate (k) for this project is 13%.

$$NPV = \frac{\$10,000}{(1.13)^1} + \frac{\$12,000}{(1.13)^2} + \frac{\$15,000}{(1.13)^3} + \frac{\$10,000}{(1.13)^4} + \frac{\$7,000}{(1.13)^5} - \$40,000$$

NPV Solution

NPV = $\$10,000(PVIF_{13\%,1}) + \$12,000(PVIF_{13\%,2})$
+ $\$15,000(PVIF_{13\%,3}) + \$10,000(PVIF_{13\%,4})$
+ $\$\ 7,000(PVIF_{13\%,5}) - \$40,000$

NPV = \$10,000(.885) + \$12,000(.783) +
\$15,000(.693) + \$10,000(.613) +
\$ 7,000(.543) - \$40,000

NPV = \$8,850 + \$9,396 + \$10,395 +
\$6,130 + \$3,801 - \$40,000

= - \$1,428

NPV Acceptance Criterion

The management of *Sankar Baskets* has determined that the required rate is 13% for projects of this type.

Should this project be accepted?

No! The NPV is <u>negative</u>. This means that the project is reducing shareholder wealth. [*Reject* as $NPV < 0$]

NPV Strengths and Weaknesses

Strengths:

- Cash flows assumed to be reinvested at the hurdle rate.
- Accounts for TVM.
- Considers all cash flows.

Profitability Index (PI)

PI is the ratio of the present value of a project's future net cash flows to the project's initial cash outflow.

$$PI = \left[\frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots + \frac{CF_n}{(1+k)^n} \right] \div ICO$$

PI Acceptance Criterion

PI = $38,572 / $40,000

= .9643 *(Method #1, 13-33)*

Should this project be accepted?

No! The PI is <u>less than 1.00</u>. This means that the project is not profitable. [*Reject* as *PI* < *1.00*]

PI Strengths and Weaknesses

Strengths:

- Same as NPV
- Allows comparison of different scale projects

Weaknesses:

- Same as NPV
- Provides only relative profitability
- Potential Ranking Problems

Other Project Relationships

- ***Dependent*** -- A project whose acceptance depends on the acceptance of one or more other projects.

Potential Problems Under Mutual Exclusivity

Ranking of project proposals *may* create contradictory results.

A. Scale of Investment
B. Cash-flow Pattern
C. Project Life

A. Scale Differences

Compare a small (S) and a large (L) project.

	NET CASH FLOWS	
END OF YEAR	Project S	Project L
0	-$100	-$100,000
1	0	0
2	$400	$156,250

Scale Differences

Calculate the PBP, IRR, NPV@10%, and PI@10%.

Which project is preferred? Why?

Project	IRR	NPV	PI
S	100%	$ 231	3.31
L	25%	$29,132	1.29

B. Cash Flow Pattern

Let us compare a *decreasing* cash-flow (D) project and an *increasing* cash-flow (I) project.

END OF YEAR	NET CASH FLOWS Project D	Project I
0	-$1,200	-$1,200
1	1,000	100
2	500	600
3	100	1,080

Cash Flow Pattern

Calculate the IRR, NPV@10%, and PI@10%.

Which project is preferred?

Project	IRR	NPV	PI
D	23%	$198	1.17
I	17%	$198	1.17

C. Project Life Differences

Let us compare a *long* life (X) project and a *short* life (Y) project.

	NET CASH FLOWS	
END OF YEAR	**Project X**	**Project Y**
0	-$1,000	-$1,000
1	0	2,000
2	0	0
3	3,375	0

Project Life Differences

Calculate the PBP, IRR, NPV@10%, and PI@10%.

Which project is preferred? Why?

Project	IRR	NPV	PI
X	50%	$1,536	2.54
Y	100%	$ 818	1.82

Another Way to Look at Things

1. Adjust cash flows to a common terminal year if project "Y" will ***NOT*** be replaced.

Compound Project Y, Year 1 @10% for 2 years.

Year	0	1	2	3
CF	-$1,000	$0	$0	$2,420

Results: IRR* = 34.26% NPV = $818

**Lower IRR* from adjusted cash-flow stream. X is still Best.

Replacing Projects with Identical Projects

2. Use *Replacement Chain Approach* when project "Y" will be replaced.

0	1	2	3
-$1,000	$2,000		
	-1,000	$2,000	
		-1,000	$2,000
-$1,000	$1,000	$1,000	$2,000

Results: **IRR* = 100%** **NPV* = $2,238.17**

Higher NPV, but the same IRR. Y is Best.

Capital Rationing

Capital Rationing occurs when a constraint (or budget ceiling) is placed on the total size of capital expenditures during a particular period.

Example: Sameer must determine what investment opportunities to undertake for *Sameer Baskets (SB)*. He is limited to a maximum expenditure of $32,500 *only* for this capital budgeting period.

Available Projects for SB

Project	ICO	IRR	NPV	PI
A	$ 500	18%	$ 50	1.10
B	5,000	25	6,500	2.30
C	5,000	37	5,500	2.10
D	7,500	20	5,000	1.67
E	12,500	26	500	1.04
F	15,000	28	21,000	2.40
G	17,500	19	7,500	1.43
H	25,000	15	6,000	1.24

Choosing by IRRs for SB

Project	ICO	IRR	NPV	PI
C	$ 5,000	37%	$ 5,500	2.10
F	15,000	28	21,000	2.40
E	12,500	26	500	1.04
B	5,000	25	6,500	2.30

Projects C, F, and E have the three *largest IRRs*.

The resulting *increase* in shareholder wealth is $27,000 with a $32,500 outlay.

Choosing by NPVs for SB

Project	ICO	IRR	NPV	PI
F	$15,000	28%	$21,000	2.40
G	17,500	19	7,500	1.43
B	5,000	25	6,500	2.30

Projects F and G have the two *largest NPVs*.

The resulting *increase* in shareholder wealth is $28,500 with a $32,500 outlay.

Choosing by PIs for SB

Project	ICO	IRR	NPV	PI
F	$15,000	28%	$21,000	2.40
B	5,000	25	6,500	2.30
C	5,000	37	5,500	2.10
D	7,500	20	5,000	1.67
G	17,500	19	7,500	1.43

Projects F, B, C, and D have the four *largest PIs*.

The resulting *increase* in shareholder wealth is $38,000 with a $32,500 outlay.

Summary of Comparison

Method	Projects Accepted	Value Added
PI	F, B, C, and D	$38,000
NPV	F and G	$28,500
IRR	C, F, and E	$27,000

PI generates the *greatest increase* in *shareholder wealth* when a limited capital budget exists for a *single period*.

Post-Completion Audit

Post-completion Audit

A formal comparison of the actual costs and benefits of a project with original estimates.

- Identify any project weaknesses
- Develop a possible set of corrective actions
- Provide appropriate feedback

Result: Making better future decisions!

Multiple IRR Problem*

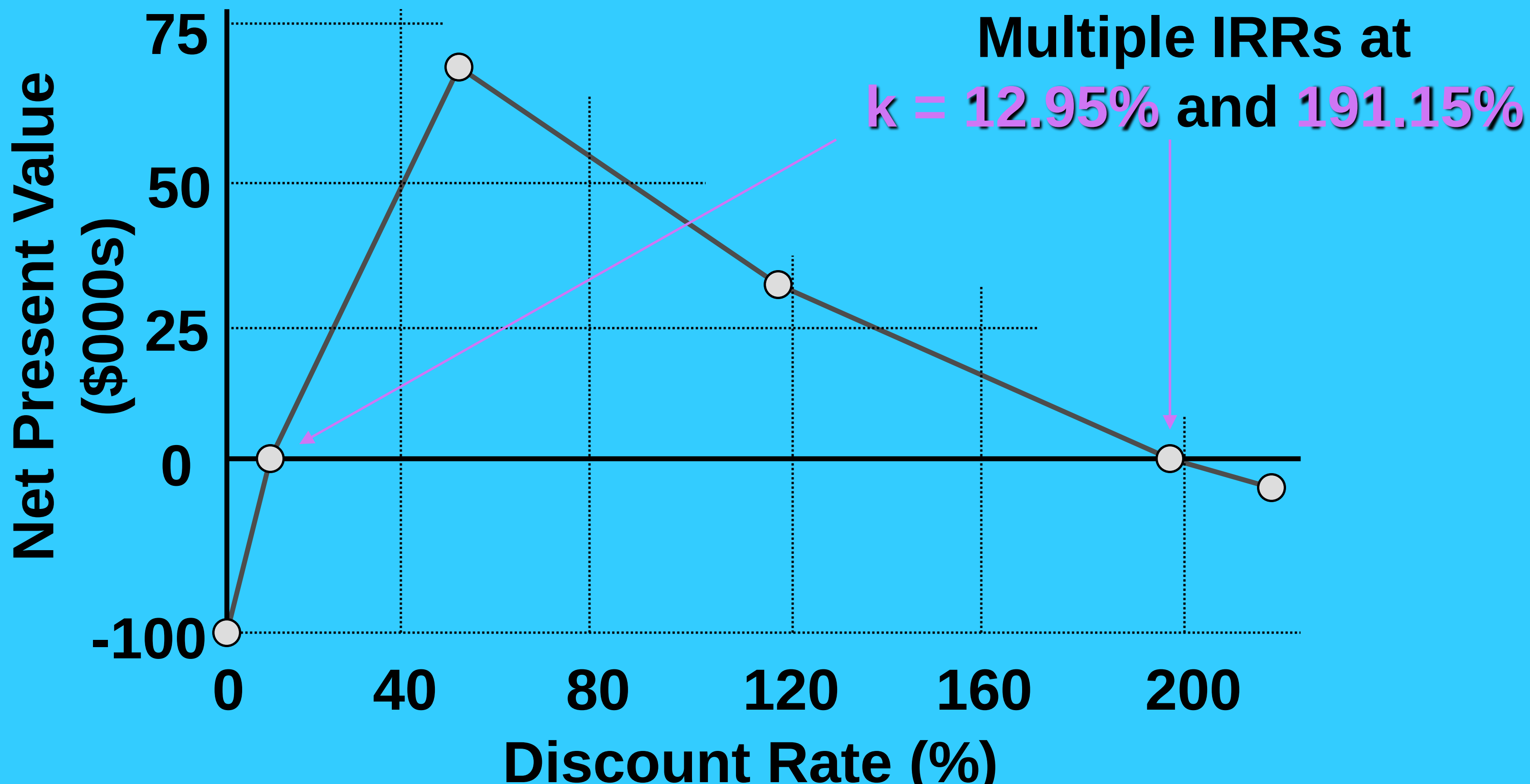
Let us assume the following cash flow pattern for a project for Years 0 to 4:

-$100 +$100 +$900 -$1,000

How many *potential* IRRs could this project have?

Two!! There are as many potential IRRs as there are sign changes.

NPV Profile -- Multiple IRRs



Risk and Managerial Options in Capital Budgeting

Risk and Managerial Options in Capital Budgeting

- The Problem of Project Risk
- Total Project Risk
- Contribution to Total Firm Risk: Firm-Portfolio Approach
- Managerial Options

An Illustration of Total Risk (Discrete Distribution)

ANNUAL CASH FLOWS: YEAR 1

PROPOSAL A

State	Probability	Cash Flow
Deep Recession	.05	$ -3,000
Mild Recession	.25	1,000
Normal	.40	5,000
Minor Boom	.25	9,000
Major Boom	.05	13,000

Summary of Proposal A

The standard deviation = SQRT (14,400,000) = $3,795

The expected cash flow = $5,000

An Illustration of Total Risk (Discrete Distribution)

ANNUAL CASH FLOWS: YEAR 1

PROPOSAL B

State	Probability	Cash Flow
Deep Recession	.05	$ -1,000
Mild Recession	.25	2,000
Normal	.40	5,000
Minor Boom	.25	8,000
Major Boom	.05	11,000

Summary of Proposal B

The standard deviation = SQRT (8,100,000) = **$2,846**

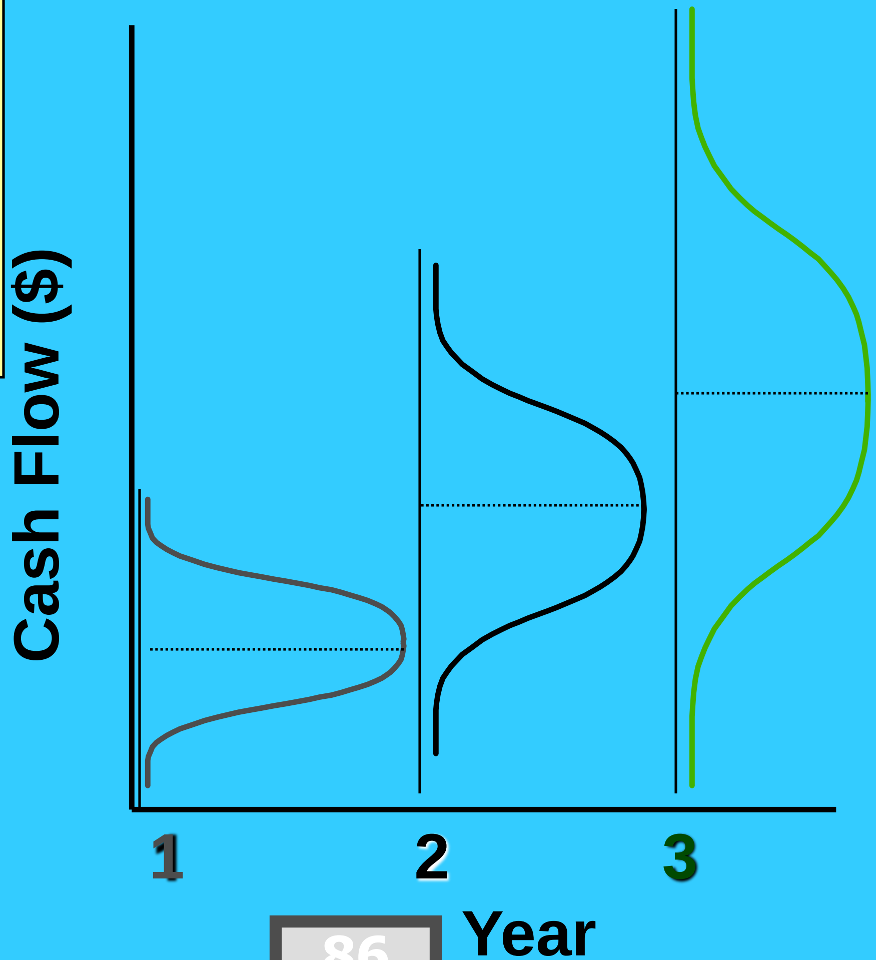
The expected cash flow = **$5,000**

The standard deviation of Proposal B < Proposal A.
($2,846 < $3,795)

Total Project Risk

Projects have risk that may *change* from period to period.

Projects are more likely to have *continuous*, rather than discrete distributions.



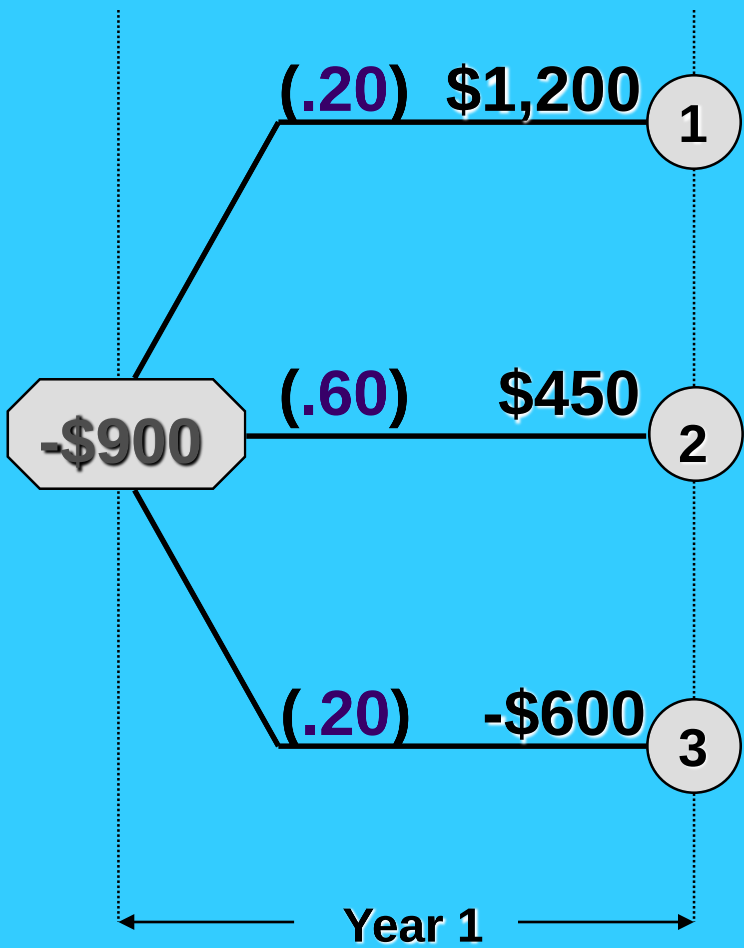
Probability Tree Approach

A graphic or tabular approach for organizing the possible cash-flow streams generated by an investment. The presentation resembles the branches of a tree. Each complete branch represents one possible cash-flow sequence.

Probability Tree Approach

Basket Wonders is examining a project that will have an initial cost today of $900. Uncertainty surrounding the first year cash flows creates three possible cash-flow scenarios in Year 1.

Probability Tree Approach

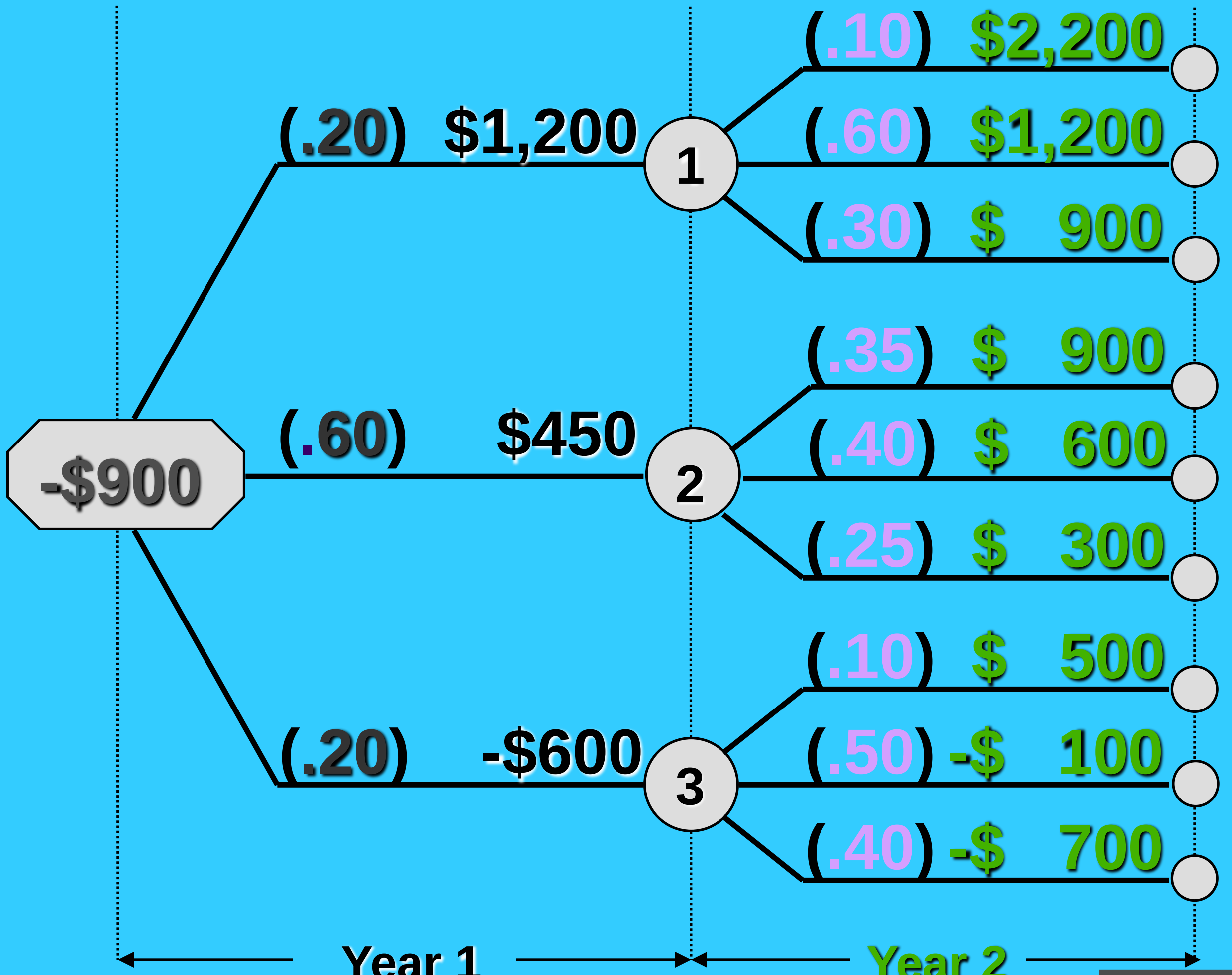


Node 1: 20% chance of a $1,200 cash-flow.

Node 2: 60% chance of a $450 cash-flow.

Node 3: 20% chance of a - $600 cash-flow.

Probability Tree Approach

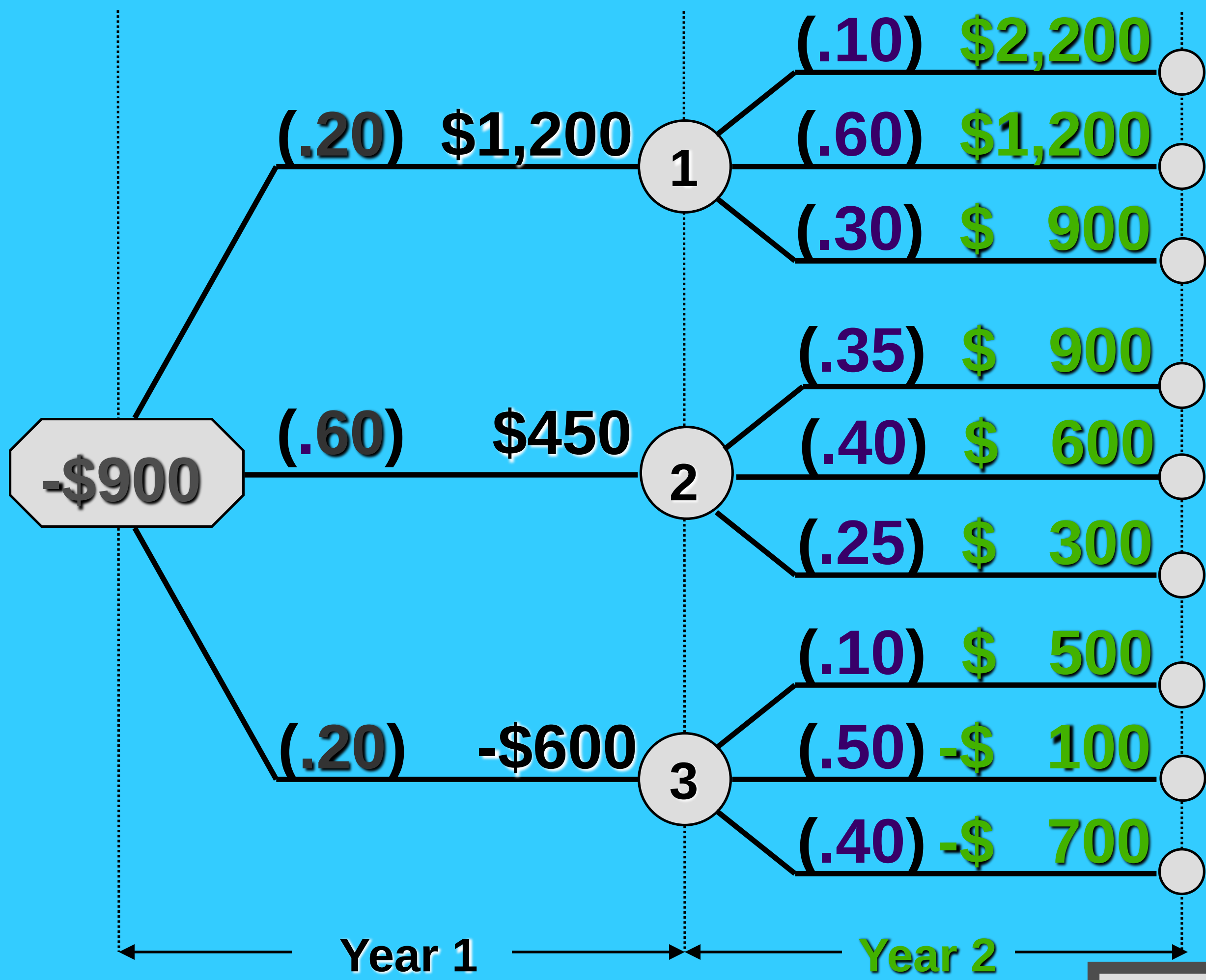


Each node in Year 2 represents a *branch* of our probability tree.

The probabilities are said to be conditional probabilities.

2009

Joint Probabilities [P(1,2)]



.02	Branch 1
.12	Branch 2
.06	Branch 3
.21	Branch 4
.24	Branch 5
.15	Branch 6
.02	Branch 7
.10	Branch 8
.08	Branch 9

Project NPV Based on Probability Tree Usage

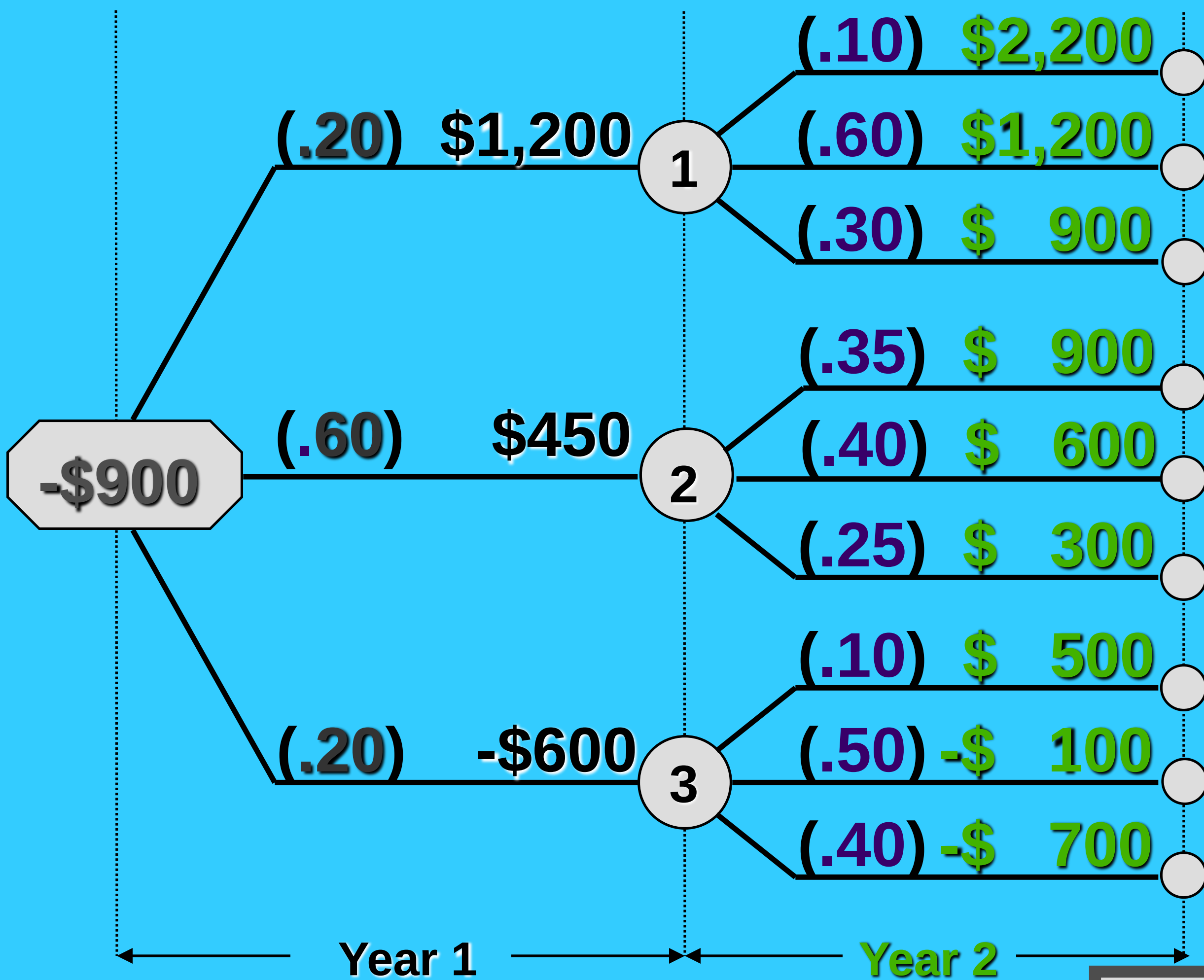
The probability tree accounts for the distribution of cash flows. Therefore, discount all cash flows at *only* the risk-free rate of return.

$$\overline{NPV} = \sum_{i=1}^{z} (NPV_i)(P_i)$$

The NPV for branch i of the probability tree for two years of cash flows is

$$NPV_i = \frac{CF_1}{(1 + R_f)^1} + \frac{CF_2}{(1 + R_f)^2} - ICO$$

NPV for Each Cash-Flow Stream at 5% Risk-Free Rate



$ 2,238.32
$ 1,331.29
$ 1,059.18

$ 344.90
$ 72.79
-$ 199.32

-$ 1,017.91
-$ 1,562.13
-$ 2,106.35

Calculating the Expected Net Present Value (NPV)

Branch	NPV_i	P(1,2)	NPV_i * P(1,2)
Branch 1	$ 2,238.32	.02	$ 44.77
Branch 2	$ 1,331.29	.12	$159.75
Branch 3	$ 1,059.18	.06	$ 63.55
Branch 4	$ 344.90	.21	$ 72.43
Branch 5	$ 72.79	.24	$ 17.47
Branch 6	-$ 199.32	.15	-$ 29.90
Branch 7	-$ 1,017.91	.02	-$ 20.36
Branch 8	-$ 1,562.13	.10	-$156.21
Branch 9	-$ 2,106.35	.08	-$168.51

Expected Net Present Value = -$ 17.01

Summary of the Decision Tree Analysis

The standard deviation = SQRT ($1,031,800) = $1,015.78

The expected NPV = -$ 17.01

Simulation Approach

An approach that allows us to test the possible results of an investment proposal before it is accepted. Testing is based on a model coupled with probabilistic information.

Simulation Approach

Each proposal will generate an internal rate of return. The process of generating many, many simulations results in a large set of internal rates of return. The distribution might look like the following: