Time Value of Money

Compiled from Van Horne & Wachowicz

Which would you prefer – Rs.10,000 today or Rs.10,000 after 2 years?

Obviously, Rs.10,000 today.

This is *TIME VALUE TO MONEY!!*

Types of Interest

- **Simple Interest**

  Interest paid (earned) on only the original amount, or principal borrowed (lent).

- Compound Interest

  Interest paid (earned) on any previous interest earned, as well as on the principal borrowed (lent).

Simple Interest Formula

Formula

$$SI = P_0(i)(n)$$

SI: Simple Interest

P_0: Deposit today (t=0)

i: Interest Rate per Period

n: Number of Time Periods

Simple Interest Example

- Assume that you deposit Rs.1,000 in an account earning 7% simple interest for 2 years. *What is the accumulated interest at the end of the 2nd year?*

- SI $= P_0(i)(n)$
  $= Rs.1,000(.07)(2)$
  $= Rs.140$

Simple Interest (FV)

- What is the Future Value (FV) of the deposit?

$$FV = P_0 + SI$$
$$= Rs.1,000 + Rs.140$$
$$= Rs.1,140$$

- Future Value is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate.

Simple Interest (PV)

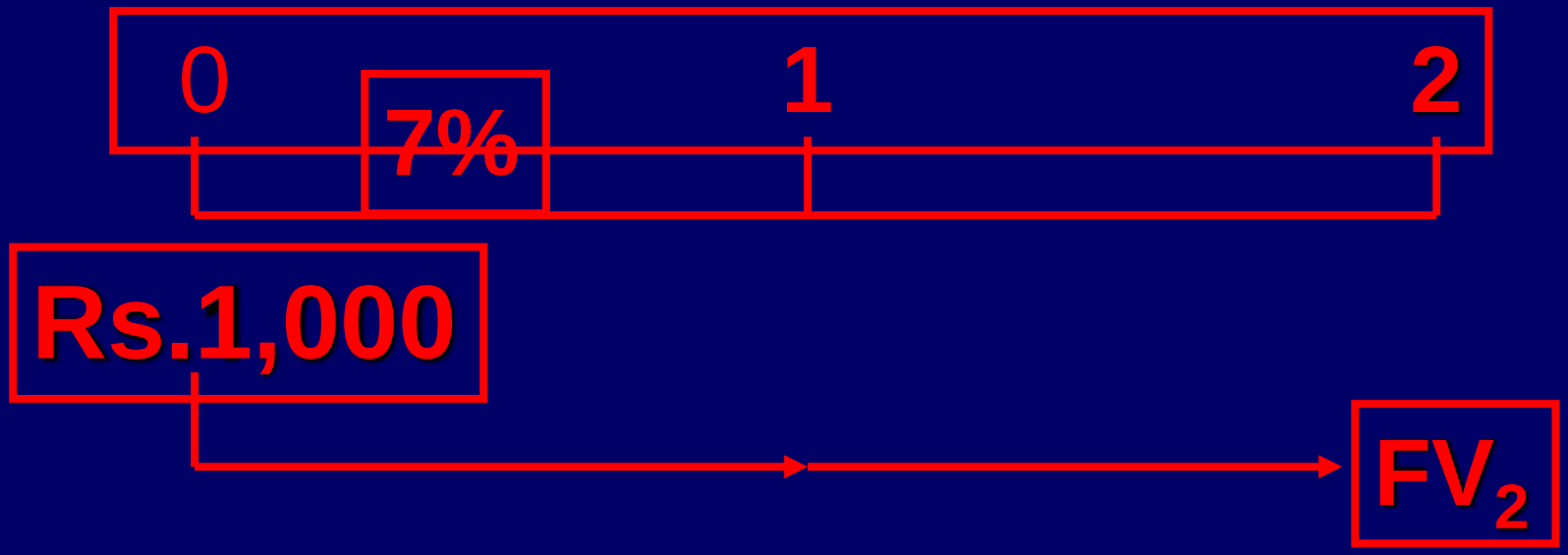
- What is the Present Value (PV) of the previous problem?

  The Present Value is simply the Rs.1,000 you originally deposited. That is the value today!

- Present Value is the current value of a future amount of money, or a series of payments, evaluated at a given interest rate.

Future Value Single Deposit

Assume that you deposit Rs.1,000 at a compound interest rate of 7% for 2 years.



Future Value Single Deposit

$FV_1 = P_0 (1+i)^1 = Rs.1,000\ (1.07)$

$= Rs.1,070$

$FV_2 = P_0 (1+i)(1+i) = Rs.1,000(1.07)(1.07)$

$= P_0 (1+i)^2 = Rs.1,000(1.07)^2 = Rs.1,144.90$

You earned an *EXTRA Rs.4.90* in Year 2 with compound over simple interest.

General Future Value Formula

$$FV_1 = P_0(1+i)^1$$
$$FV_2 = P_0(1+i)^2$$

General Future Value Formula:

$$FV_n = P_0 (1+i)^n$$

or $$FV_n = P_0 (FVIF_{i,n})$$

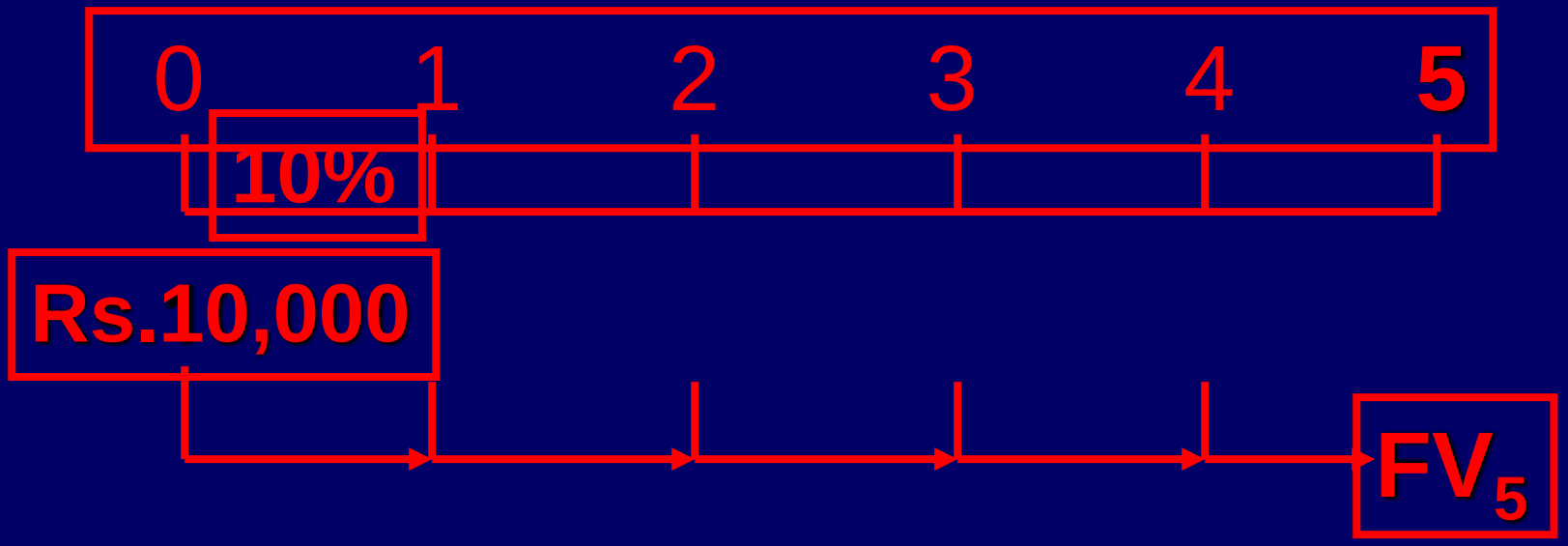
Using Future Value Tables

FV_2 = Rs.1,000 ($FVIF_{7\%,2}$)
= Rs.1,000 (1.145)
= **Rs.1,145** [Due to Rounding]

Period	6%	7%	8%
1	1.060	1.070	1.080
2	1.124	**1.145**	1.166
3	1.191	1.225	1.260
4	1.262	1.311	1.360
5	1.338	1.403	1.469

Example

Aditi wants to know how large her deposit of Rs.10,000 today will become at a compound annual interest rate of 10% for 5 years.



Solution

- **Calculation based on general formula:**

$$FV_n = P_0 (1+i)^n$$

$$FV_5 = Rs.10,000 (1+ 0.10)^5$$

$$= Rs.16,105.10$$

- Calculation based on Table I:

$$FV_5 = Rs.10,000 (FVIF_{10\%, 5})$$

$$= Rs.10,000 (1.611)$$

$$= Rs.16,110$$ [*Due to Rounding*]

Quick ...

How long does it take to double Rs.5,000 at a compound rate of 12% per year (approx.)?

Approx. Years to Double = 72 / i%

72 / 12% = *6 Years*

[Actual Time is 6.12 Years]

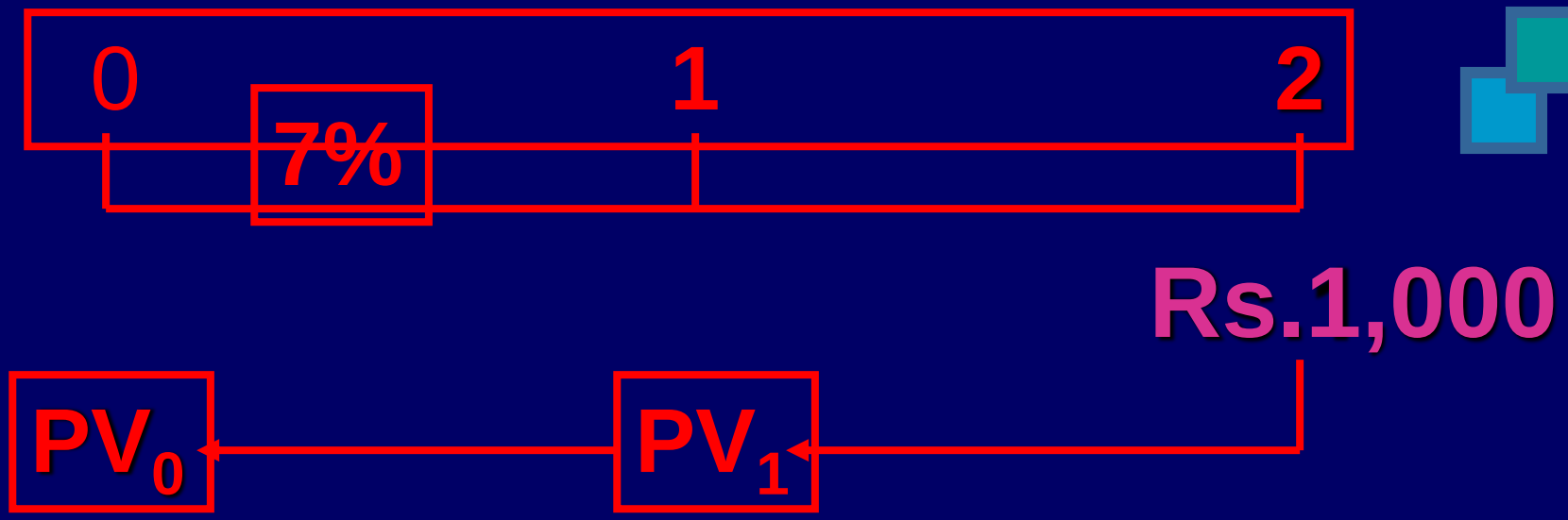
Usage …

- On short-term instruments, interest is usually ‘simple’ rather than ‘compound’.
- Ex: If the investor places Rs. 73 lakhs on t-bill at 8% for 92 days, he will receive:

Rs.73 lakhs $\times$ (1+ (0.08 $\times$ 92/365))

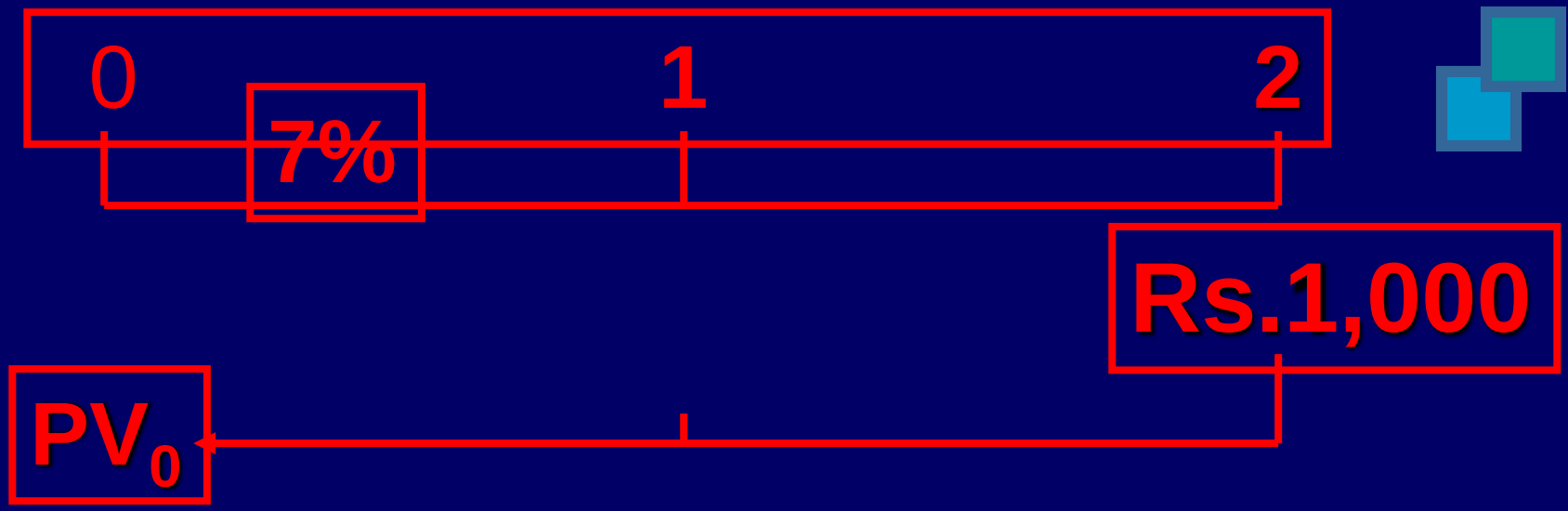
Present Value Single Deposit

Assume that you need Rs.1,000 in 2 years. Let's examine the process to determine how much you need to deposit today at a discount rate of 7% compounded annually.



Present Value Single Deposit

$PV_0 = FV_2 / (1+i)^2 = Rs.1,000 / (1.07)^2 = FV_2 / (1+i)^2 = Rs.873.44$



General Present Value Formula

$$PV_0 = FV_1 / (1+i)^1$$

$$PV_0 = FV_2 / (1+i)^2$$

General Present Value Formula:

$$PV_0 = FV_n / (1+i)^n$$

or $$PV_0 = FV_n (PVIF_{i,n})$$

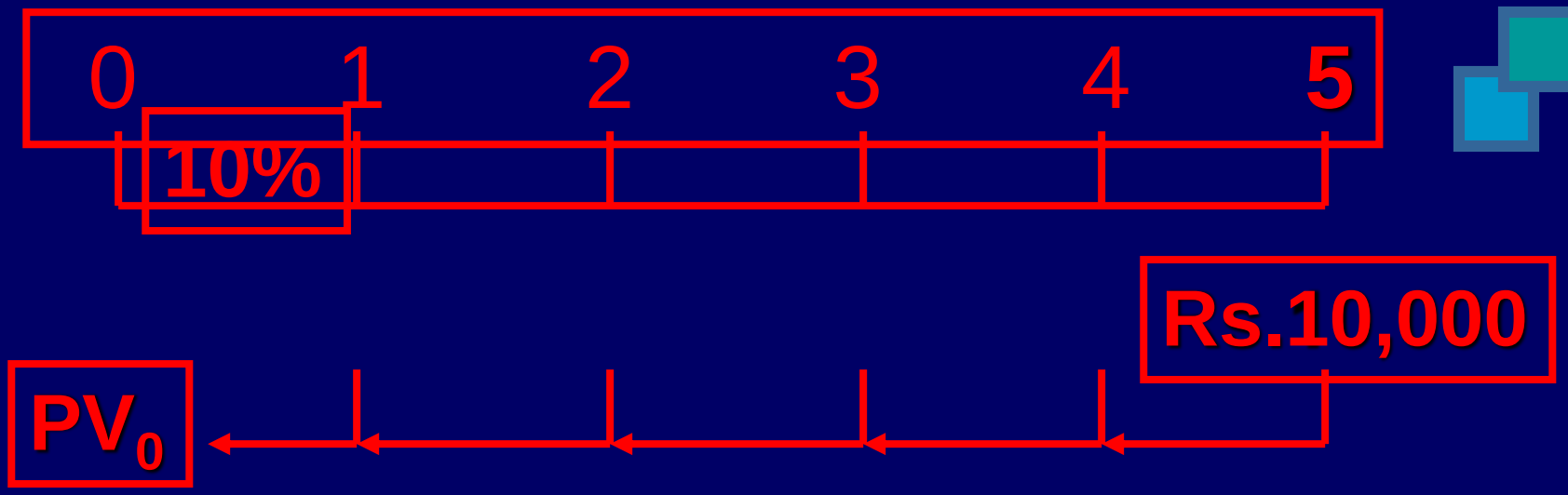
Using Present Value Tables

PV_2 = Rs.1,000 $(PVIF_{7\%,2})$
= Rs.1,000 (.873)
= **Rs.873** [Due to Rounding]

Period	6%	7%	8%
1	.943	.935	.926
2	.890	**.873**	.857
3	.840	.816	.794
4	.792	.763	.735
5	.747	.713	.681

Example

Aneesh wants to know how large of a deposit to make so that the money will grow to Rs.10,000 in 5 years at a discount rate of 10%.



Solution

Calculation based on general formula:

$PV_0 = FV_n / (1+i)^n$

$PV_0 = Rs.10,000 / (1+ 0.10)^5$

$= Rs.6,209.21$

Calculation based on Table:

$PV_0 = Rs.10,000\ (PVIF_{10\%,\ 5})$

$= Rs.10,000\ (.621)$

$= Rs.6,210.00$ [*Due to Rounding*]

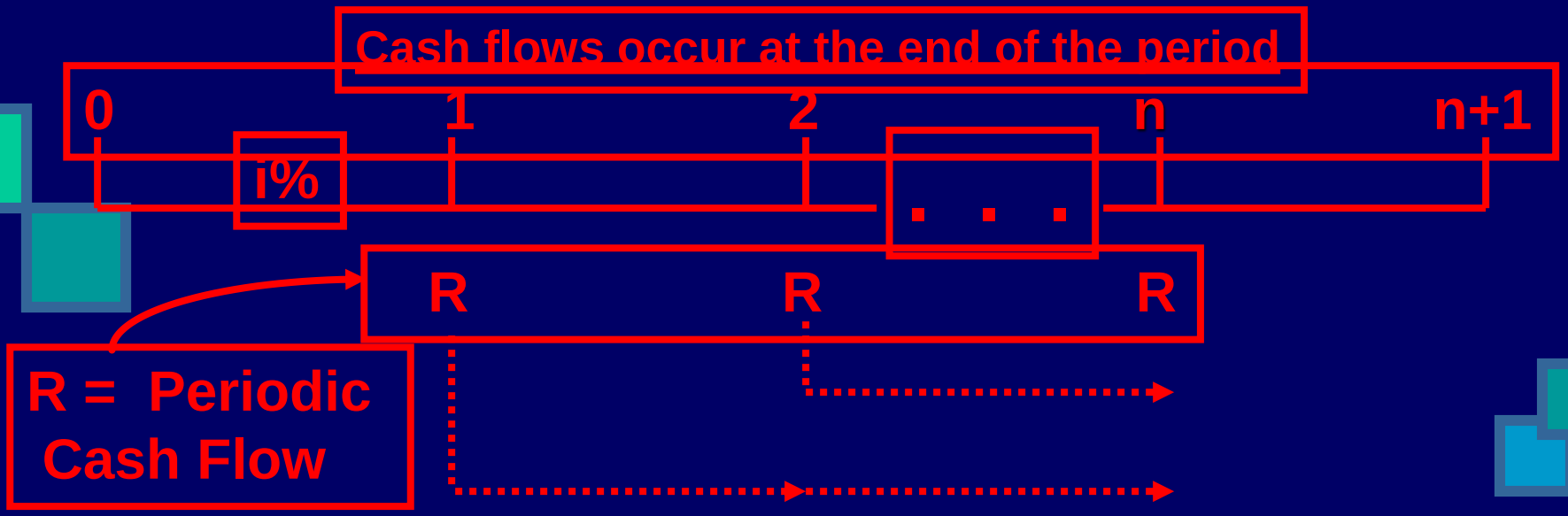
Types of Annuities

- *An Annuity* represents a series of equal payments (or receipts) occurring over a specified number of equidistant periods.

Examples of Annuities

- Student Loan Payments
- Car Loan Payments
- Insurance Premiums
- Recurring Deposits
- Retirement Savings

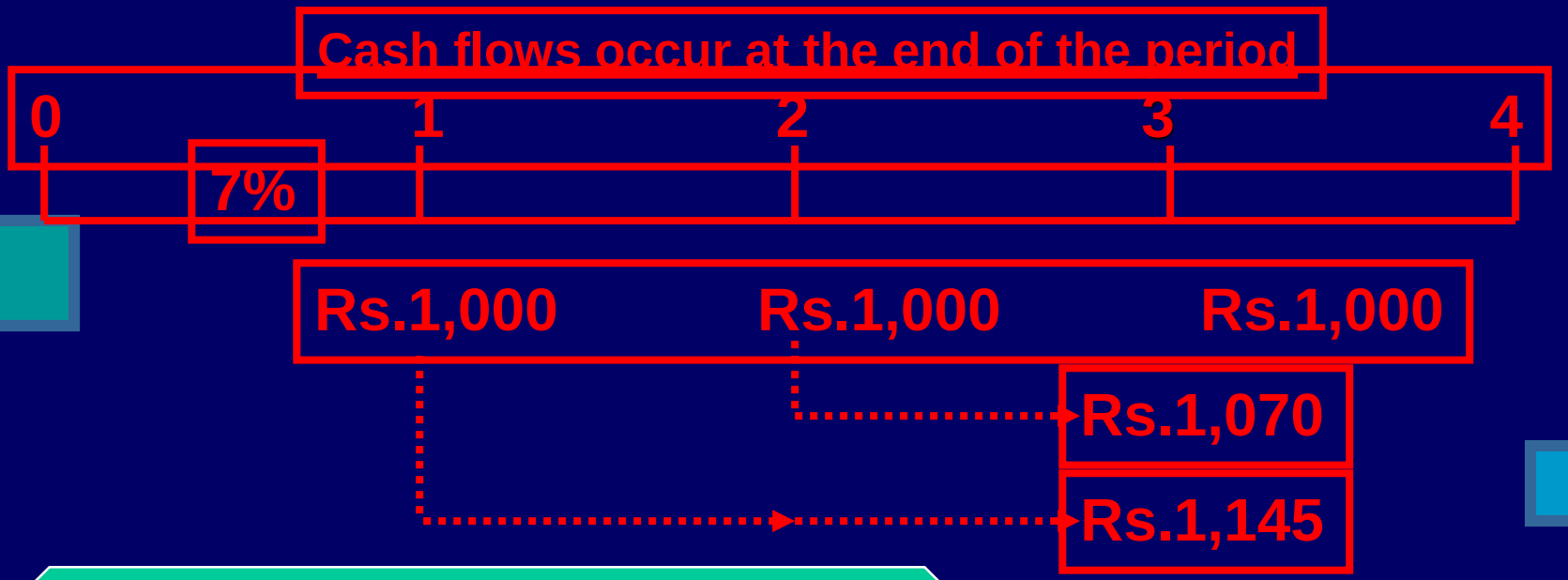
Overview of an Ordinary Annuity -- FVA



$FVA_n = R(1+i)^{n-1} + R(1+i)^{n-2} + \ldots + R(1+i)^1 + R(1+i)^0$

FVA_n

Example of an Ordinary Annuity -- FVA



$FVA_3 = Rs.1,000(1.07)^2 + Rs.1,000(1.07)^1 + Rs.1,000(1.07)^0$

$= Rs.1,145 + Rs.1,070 + Rs.1,000$

$= Rs.3,215$

Rs.3,215 = FVA_3

Valuation Using Tables

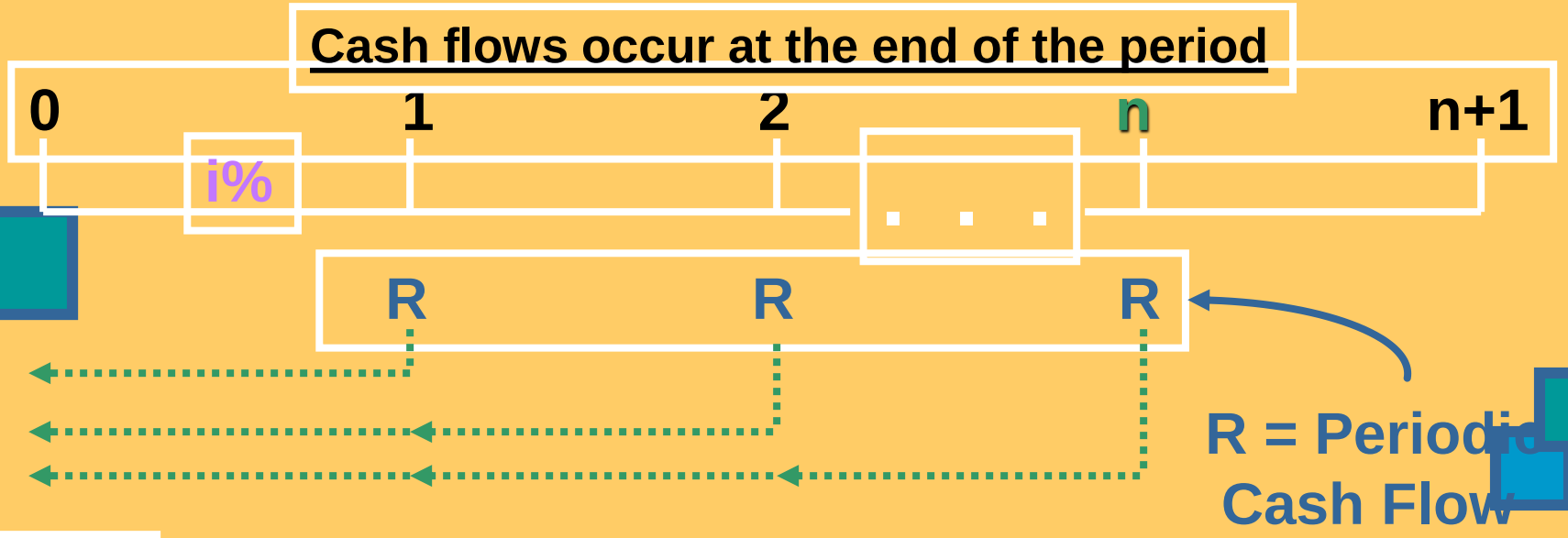
$FVA_n = R\ (FVIFA_{i\%,n})$

$FVA_3 = Rs.1{,}000\ (FVIFA_{7\%,3})$

$= Rs.1{,}000\ (3.215) = Rs.3{,}215$

Period	6%	7%	8%
1	1.000	1.000	1.000
2	2.060	2.070	2.080
3	3.184	**3.215**	3.246
4	4.375	4.440	4.506
5	5.637	5.751	5.867

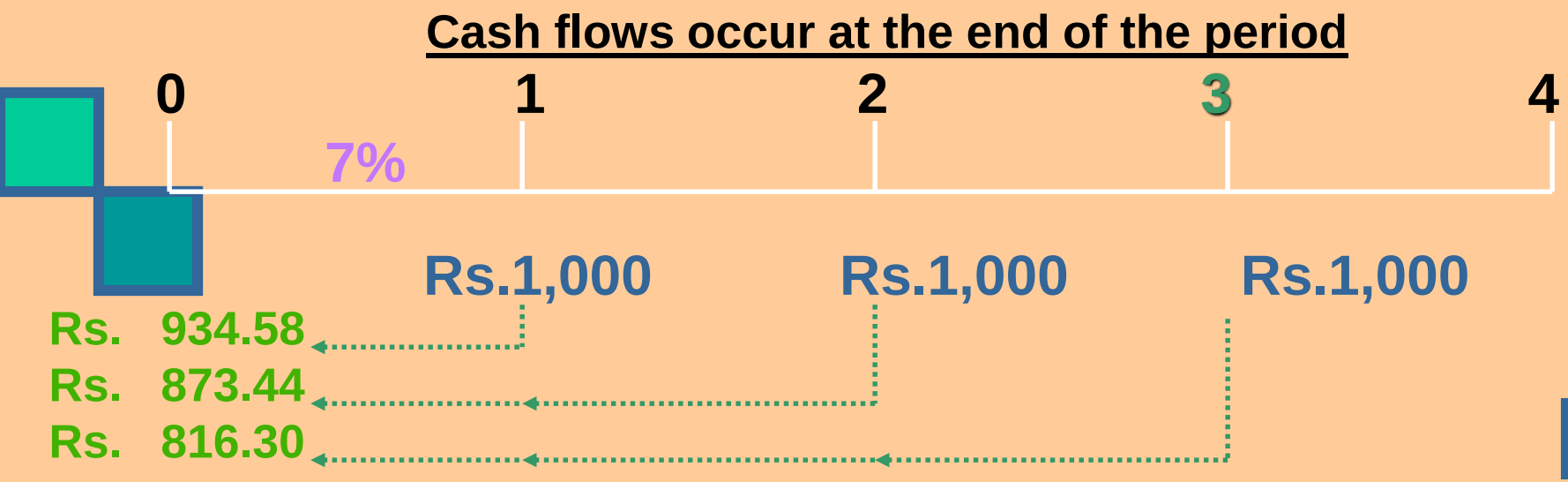
Overview of an Ordinary Annuity -- PVA



PVA_n

$$PVA_n = R/(1+i)^1 + R/(1+i)^2 + \ldots + R/(1+i)^n$$

Example of an Ordinary Annuity -- PVA



Rs.2,624.32 = PVA_3

PVA_3 = Rs.1,000/$(1.07)^1$ + Rs.1,000/$(1.07)^2$ + Rs.1,000/$(1.07)^3$

= Rs.934.58 + Rs.873.44 + Rs.816.30

= Rs.2,624.32

Valuation Using Tables

$PVA_n = R\ (PVIFA_{i\%,n})$

$PVA_3 = Rs.1{,}000\ (PVIFA_{7\%,3})$

$= Rs.1{,}000\ (2.624) = Rs.2{,}624$

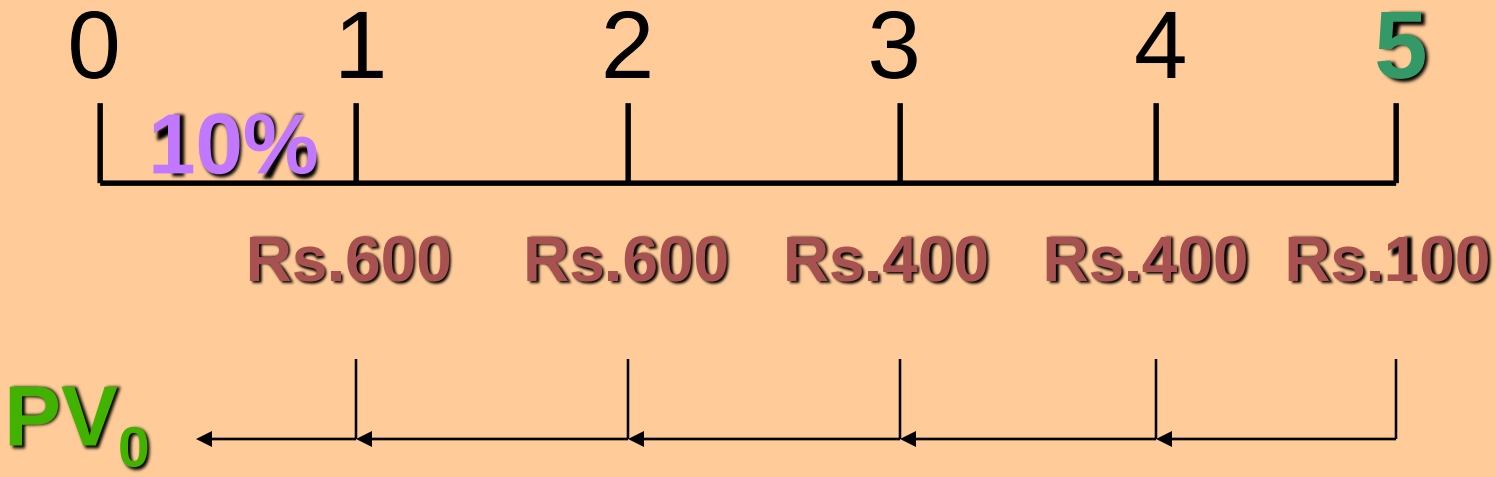
Period	6%	7%	8%
1	0.943	0.935	0.926
2	1.833	1.808	1.783
3	2.673	**2.624**	2.577
4	3.465	3.387	3.312
5	4.212	4.100	3.993

Steps to Solve Time Value of Money Problems

1. Read problem thoroughly
2. Determine if it is a PV or FV problem
3. Create a time line
4. Put cash flows and arrows on time line
5. Determine if solution involves a single CF, annuity stream(s), or mixed flow
6. Solve the problem
7. Recheck your calculations (optional)

Mixed Flows Example

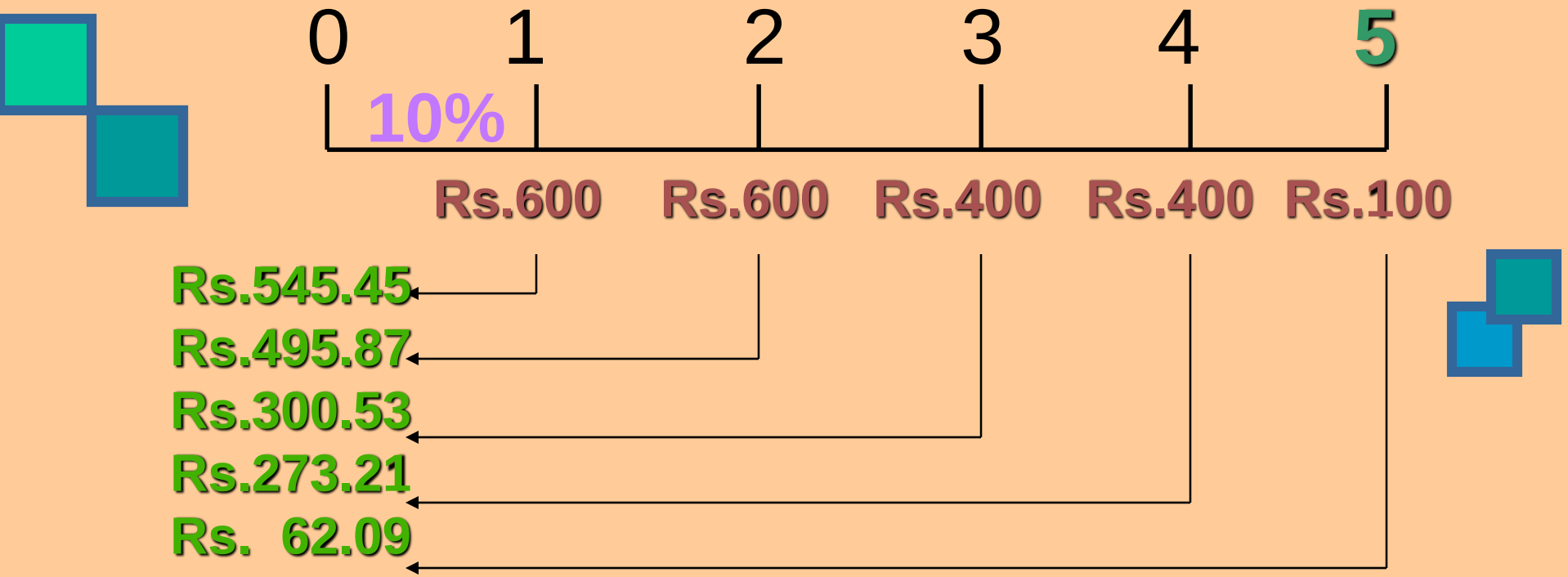
Nakul will receive the set of cash flows below. What is the **Present Value** at a discount rate of **10%**?



How to Solve?

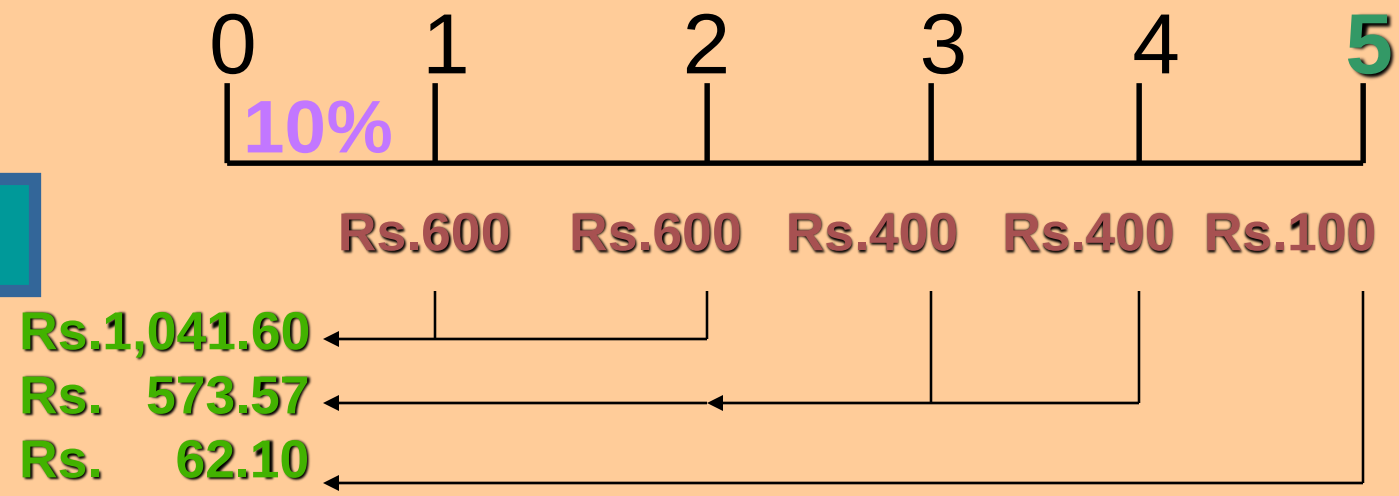
1. Solve a "*piece-at-a-time*" by discounting each *piece* back to t=0.
2. Solve a "*group-at-a-time*" by first breaking problem into groups of annuity streams and any single cash flow group. Then discount each *group* back to t=0.

"Piece-At-A-Time"



Rs.1677.15 = PV_0 of the Mixed Flow

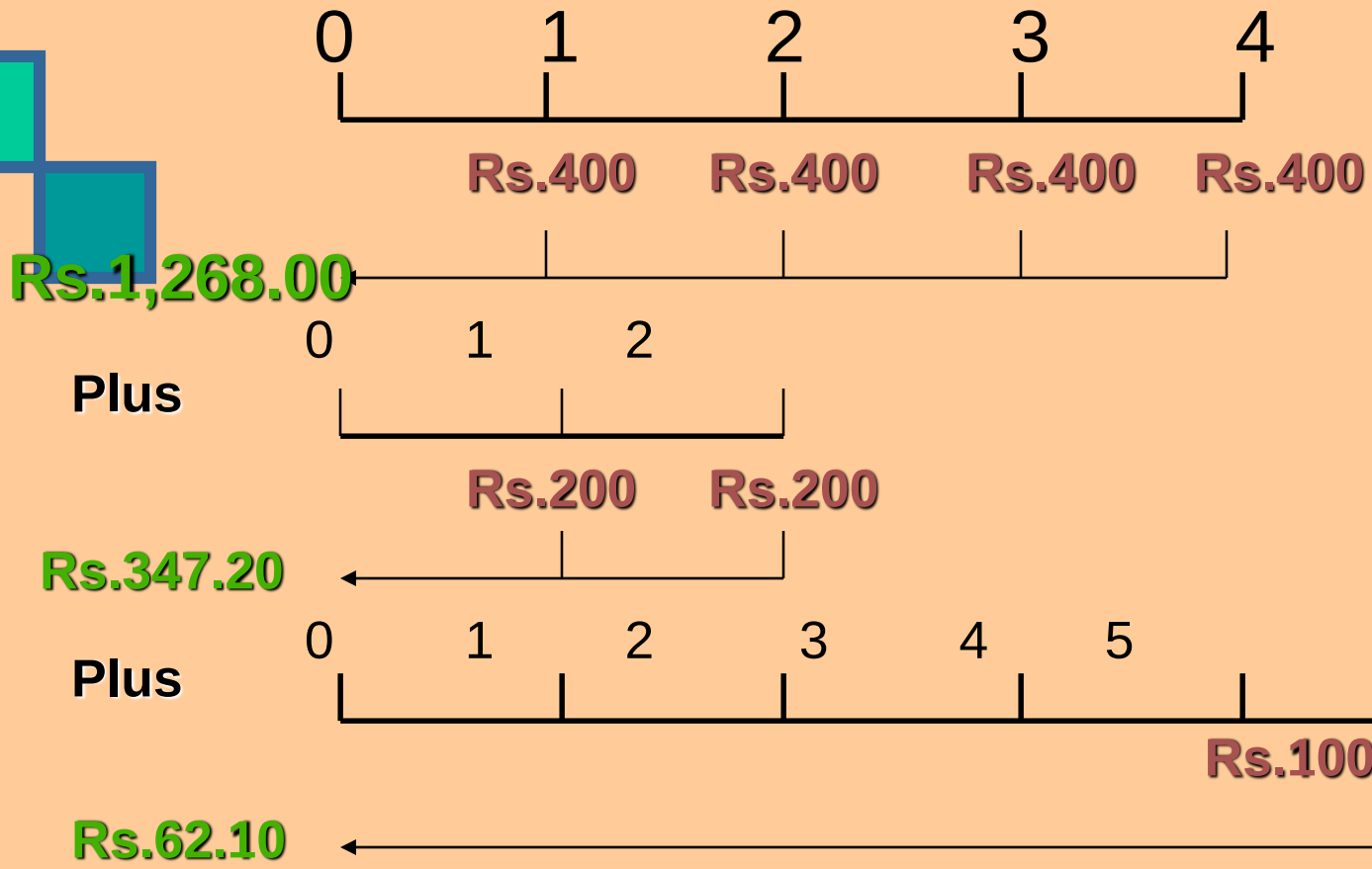
"Group-At-A-Time" (#1)



Rs.1,677.27 = PV_0 of Mixed Flow [Using Tables]

$$Rs.600(PVIFA_{10\%,2}) = Rs.600(1.736) = Rs.1,041.60$$
$$Rs.400(PVIFA_{10\%,2})(PVIF_{10\%,2}) = Rs.400(1.736)(0.826) = Rs.573.57$$
$$Rs.100\ (PVIF_{10\%,5}) = Rs.100\ (0.621) = Rs.62.10$$

"Group-At-A-Time" (#2)



PV_0 equals Rs.1677.30.

Frequency of Compounding

General Formula:

$$FV_n = PV_0(1 + [i/m])^{mn}$$

n: Number of Years

m: Compounding Periods per Year

i: Annual Interest Rate

$FV_{n,m}$: FV at the end of Year n

PV_0: PV of the Cash Flow today

Impact of Frequency

Himanshu has Rs.1,000 to invest for 2 years at an annual interest rate of 12% paid twice a year

Annual $FV_2 = 1{,}000(1+ [.12/1])^{(1)(2)}$

$= 1{,}254.40$

Semi $FV_2 = 1{,}000(1+ [.12/2])^{(2)(2)}$

$= 1{,}262.48$

Impact of Frequency

Qurtly $FV_2 = 1{,}000(1+ [.12/4])^{(4)(2)}$
$= 1{,}266.77$

Monthly $FV_2 = 1{,}000(1+ [.12/12])^{(12)(2)}$
$= 1{,}269.73$

Daily $FV_2 = 1{,}000(1+[.12/365])^{(365)(2)}$
$= 1{,}271.20$

Effective Annual Interest Rate

Effective Annual Interest Rate

The actual rate of interest earned (paid) after adjusting the *nominal rate* for factors such as the number of compounding periods per year.

$$(1 + [\, i / m \,])^m - 1$$

Effective Annual Interest Rate

Ashish has a Rs.1,000 FD at the Sahara Investments. The interest rate is 6% compounded quarterly for 1 year. What is the Effective Annual Interest Rate (EAIR)?

$$EAIR = (1 + 6\% / 4)^4 - 1 = 1.0614 - 1 = 6.14\%!$$

Steps to Amortizing a Loan

1. Calculate the payment per period.
2. Determine the interest in Period t.
   (Loan balance at t-1) x (i% / m)
3. Compute principal payment in Period t.
   (Payment - interest from Step 2)
4. Determine ending balance in Period t.
   (Balance - principal payment from Step 3)
5. Start again at Step 2 and repeat.

Amortizing a Loan Example

Vishwadeep is borrowing Rs.10,000 at a compound annual interest rate of 12%. Amortize the loan if annual payments are made for 5 years.

Step 1: Payment

$$PV_0 = R\ (PVIFA_{i\%,n})$$

$$Rs.10,000 = R\ (PVIFA_{12\%,5})$$

$$Rs.10,000 = R\ (3.605)$$

$$R = Rs.10,000 / 3.605 = Rs.2,774$$

Amortizing a Loan Example

End of Year	Payment	Interest	Principal	Ending Balance
0	---	---	---	$10,000
1	$2,774	$1,200	$1,574	8,426
2	2,774	1,011	1,763	6,663
3	2,774	800	1,974	4,689
4	2,774	563	2,211	2,478
5	2,775	297	2,478	0
	$13,871	$3,871	$10,000	

[Last Payment Slightly Higher Due to Rounding]

Usefulness of Amortization

1. **Determine Interest Expense** -- Interest expenses may reduce taxable income of the firm.

2. **Calculate Debt Outstanding** -- The quantity of outstanding debt may be used in financing the day-to-day activities of the firm.

Perpetuities

- With no growth

$PV_0 = R_1/i$

- With growth

$PV_0 = R_1/(i - g)$

Upon retirement 35 years from today, Ranjan would like to make his first of annual withdrawals from his perpetual savings account. He would like for the first withdrawal to be Rs. 1,80,000 and would like to be able to increase his withdrawals by 5% per year in order to allow for inflation. How much would he have to deposit today to achieve his goal if his account in SBI pays 8.5% per year?

- Perpetual Scheme: Rs. 51.43 lakhs → Rs. 3.21 lakhs
- 15 Years Period Scheme: ??

Problem 1

- Rs. 843 is invested for 3 years at 6.5% (paid annually). By the end of first year interest rates have risen to 7.0% (paid annually). By the end of the second year, rates have risen to 7.5% (paid annually). Whenever an interest payment is received, it is reinvested to the end of the 3-year period. What are the total proceeds by the end of the third year?

Continuation of Problem

- Will the result be different if the interest payments were reinvested only for one-year at a time, and then rolled over, rather than reinvested to the maturity of the original investment?
- If yes, what is the difference?

Problem 2

- What is the 3-year discount factor based on a 3-year interest rate of 8.5% compounded annually?
- What is the present value of Rs.270 in 3 years time?

NPV and IRR

- A net present value (NPV) is the net total of several present values (arising from cashflows at different future dates) added together, some of which may be positive and some negative.
- An internal rate of return (IRR) is the single interest rate (rate of discount) which is necessary to use when discounting a series of future values including an initial cashflow now, to achieve a zero NPV.

- What is the NPV of the following future cashflows, discouting at a rate of 7.5% per annum annually? (all figures in Rs. Crores)

Year 1 → + 83
Year 2 → - 10
Year 3 → +150

Answer: Rs. 189.30 crores

- What is the IRR of the following cashflows? (all figures in Rs. Crores)

Now → - 164
Year 1 → + 45
Year 2 → + 83
Year 3 → +75

Answer: 10.592%